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Local Operators in Massive Quantum Field Theories

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1 Introduction

A fundamental problem in quantum theory is to establish a connection between its local description (quantum field theory) and measurable quantities (particle masses, scattering amplitudes). In elementary particle physics one relies mostly on approximation techniques due to the non-integrable structure of the interactions. In order to gain a deeper understanding of quantum theory in general these issues are examined for integrable systems, where one hopes to gain exact relations between the two descriptions. Though many integrable theories in four dimensions are known (for a review see e.g. [1]) much more knowledge has been obtained so far for two dimensional theories, and it is thus this class of models where my investigations focus.

In the particle picture the interaction is encoded into the scattering matrix. The asymptotic states are described by a linear superposition of free one-particle states $|Z_\epsilon(\beta)\rangle$, which are characterised by the particle species ϵ and their momentum, parametrised as $p^{(0)} = m\cosh\beta$, $p^{(1)} = m\sinh\beta$ (m denotes the mass and β the rapidity). They are related through the S-matrix as

$$|Z_{\epsilon_1}(\beta_1)\dots Z_{\epsilon_n}(\beta_n)\rangle_{in} = S^{\epsilon'_1\dots\epsilon'_m}_{\epsilon_1\dots\epsilon_n}(\beta_1,\dots,\beta_n|\beta'_1\dots\beta'_m)\,|Z_{\epsilon'_1}(\beta'_1)\dots Z_{\epsilon'_m}(\beta'_m)\rangle_{out} \quad . \tag{1}$$

On the other hand the local description of a theory consists of the space of local operators $\mathcal{A} = \{\mathcal{O}_i\}$ and the set of multi-point correlation functions of them,

$$\langle 0|\mathcal{O}_1(x_1)\dots\mathcal{O}_n(x_n)|0\rangle \quad .$$

The two description are linked through the Lehmann-Symanzik-Zimmermann reduction, since the particles can be obtained as asymptotic limits of the local fields. Another connection is given through the form factors. Consider an arbitrary two-point correlation function

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$$G_{ij}(x) \; = < \mathcal{O}_i(x)\, \mathcal{O}_j(y) > \quad ,$$

of hermitian operators. Inserting the Identity between the two operators and expanding it into the base of asymptotic states, it can be expressed as an infinite series over multi-particle intermediate states,

$$\langle \mathcal{O}_i(x)\, \mathcal{O}_j(y) \rangle \; = \tag{2}$$
$$\sum_{n=0}^{\infty} \int \frac{d\beta_1 \dots d\beta_n}{(2\pi)^n} < 0|\mathcal{O}_i(x)|Z_{\epsilon_1}(\beta_1), \dots, Z_{\epsilon_n}(\beta_n) >_{\text{in}} {}^{\text{in}}< Z_{\epsilon_1}(\beta_1), \dots, Z_{\epsilon_n}(\beta_n)|\mathcal{O}_j(y)|0 >$$

The matrix elements

$$< 0|\mathcal{O}_i(0)|Z_{\epsilon_1}(\beta_1), \dots, Z_{\epsilon_n}(\beta_n) >= F_{\epsilon_1 \dots \epsilon_n}(\beta_1, \dots, \beta_n)$$

are called form factors and in the following I will try to explain how they can be used in order to establish a link between the local description and the particle picture of a theory.

2 Form Factors and the Space of Local Operators

If one considers two dimensional integrable theories many simplifying properties occur which allow to calculate many dynamical quantities exactly. The most remarkable fact is the factorisation of the S-matrix, which determines a general scattering process as a product of two-particle scattering amplitudes. Further these two-particle S-matrices are pure phase-shifts, that is the incoming and outgoing momenta are the same. This simplification allows to calculate the S-matrix exactly (see *e.g.* P. Kulish's lectures in these proceedings).

Also the form-factors can be determined exactly for integrable two dimensional systems. They obey a set of constraint equations, originating from fundamental principles of quantum theory, such as unitarity, analyticity, relativistic covariance and locality [3, 4]. The important fact is that the S-matrix is the only dynamical information needed. In the following I will discuss just two examples of form-factor equations, in order to determine their overall structure, and also to explain the solution techniques.

Since the theories here considered are defined in only one space dimension, a scattering process can be viewed as to interchange two particles on the real line,

$$Z_{\epsilon_1}(\beta_1) Z_{\epsilon_2}(\beta_2) = S_{\epsilon_1 \epsilon_2}(\beta_1 - \beta_2) Z_{\epsilon_2}(\beta_2) Z_{\epsilon_1}(\beta_1) \quad . \tag{3}$$

This exchange property will lead to a constraint equation for the form-factors

$$F^{\mathcal{O}}_{\epsilon_1\ldots\epsilon_i\epsilon_{i+1}\ldots\epsilon_n}(\beta_1,\ldots,\beta_i,\beta_{i+1},\ldots\beta_n) = \\ S_{\epsilon_i\epsilon_{i+1}}(\beta_i-\beta_{i+1})\, F^{\mathcal{O}}_{\epsilon_1\ldots\epsilon_{i+1}\epsilon_i\ldots\epsilon_n}(\beta_1,\ldots,\beta_{i+1},\beta_i,\ldots\beta_n) \quad . \tag{4}$$

Another constraint equation derives from the bound state structure of the theory under consideration. If particles Z_i, Z_j form a bound state Z_k, the corresponding two-particle scattering amplitude exhibits a pole at $\beta = iu^k_{ij}$ with the residue

$$-\,i \lim_{\beta\to iu^k_{ij}}(\beta - iu^k_{ij})S_{ij}(\beta) = (\Gamma^k_{ij})^2 \quad ; \tag{5}$$

Γ^k_{ij} is the three–particle on–shell vertex. Corresponding to this bound state the form–factor exhibits a pole with the residue

$$-i\lim_{\beta'\to\beta}(\beta'-\beta)F^{\mathcal{O}}_{\epsilon_1\ldots ij\ldots\epsilon_n}(\beta_1,\ldots,\beta'+i(\pi-u^j_{ik}),\beta-i(\pi-u^i_{jk}),\ldots,\beta_n) = \\ = \Gamma^k_{ij}F^{\mathcal{O}}_{\epsilon_1\ldots k\ldots\epsilon_n}(\beta_1,\ldots,\beta,\ldots,\beta_n) \quad . \tag{6}$$

As mentioned before, (4) and (6) are only two examples of form-factor equations. Nevertheless they are two exponents of the two categories of the constraint equations:

1. Equations with fixed n (e.g. (4)): they involve form factors with the same number of particles on both sides of the equation

2. Recursive equations (e.g.(6)): They link form factors with different particle numbers with each other - in the example above $n+1$ particle form factors to n particle form factors.

For theories with scalar particles there exists a well established solution method [3]. It consists of the ansatz

$$F_{\epsilon_1\ldots\epsilon_n}(\beta_1,\ldots,\beta_n) = Q_{\epsilon_1\ldots\epsilon_n}(e^{\beta_1},\ldots,e^{\beta_n})\prod_{i<j}^{n}F_{\epsilon_i\epsilon_j}(\beta_i-\beta_j) \quad . \tag{7}$$

The two-particle form factors $F_{\epsilon_i\epsilon_j}$ can be calculated easily from the form factor equations. The product term satisfies all equations of the first type (with fixed particle number) and also is designed in order to have the correct pole structure of an n-particle form factor.

Through this parametrisation the form-factor equations are reduced to recursive relations for the functions $Q_{\epsilon_1\ldots\epsilon_n}(e^{\beta_1},\ldots,e^{\beta_n})$. Further properties of these functions can be extracted from the form factor equations: they are homogeneous polynomials, symmetric

in repeated indices with a total degree in its arguments fixed by relativistic covariance and the partial degree determined from the recursion relations.

This information is sufficient for simple models to obtain explicit expressions for the form factors. In all cases though it is possible to determine the space of local operators by just considering these general properties of the functions Q [5]. Each linear independent solution of the form factor equations corresponds to an independent local operator. Therefore the space of local operators can be determined by counting the number of independent solutions of the form factor equations. This can be done due to the property of the recursion relations, that the dimension of the solution space at level n is the sum of the dimension of the solution space at level $n-1$ and of the dimension of the kernel of the recursion relation at level n. Symbolically this can be written as

$$dim(Q_n) = dim(Q_{n-1}) + dim(\mathcal{K}_n) \quad .$$

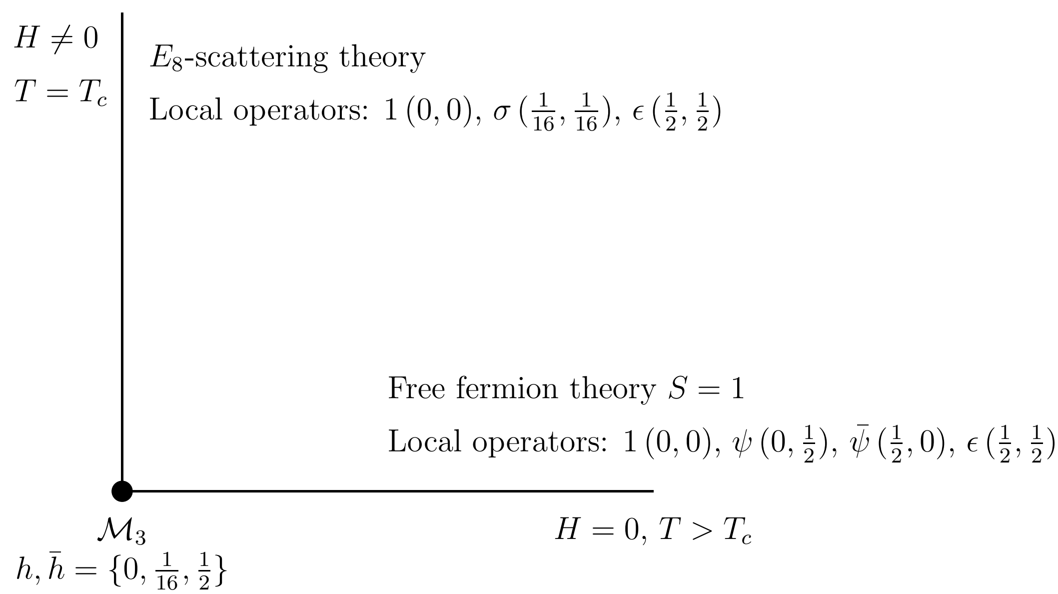

Figure 1: The critical Ising model and its integrable perturbations

An interesting application of this counting method is in perturbed conformal field theories. As an example consider the Ising model. Its point of second order phase transition is described by a conformal field theory (the minimal model $\mathcal{M}_3$) and therefore the space of local operators at the critical point is determined by Virasoro irreducible representations. The critical point admits two relevant perturbations which are integrable. The perturbation with the conformal operator with conformal weight $h = \frac{1}{2}$ drives the model into the regime $T > T_c$ and $H = 0$. It is described by a free fermion theory. The

other perturbation with the operator $h = \frac{1}{16}$ corresponds to the Ising model with $H \neq 0$ and $T = T_c$ and is described by a scattering theory containing 8 scalar massive particles [6]. Virasoro symmetry is obviously broken by both perturbations and it is therefore an interesting problem to determine the space of operators for these theories.

The situation is summarised in figure 1. For both perturbations it is possible to determine the space of local operators. It is given in terms of characters of the minimal model M_3. This is a quite remarkable fact, since conformal symmetry is explicitly broken by the perturbation. Further, since the thermal perturbation is described by a free fermion theory, the local operators are just the fermions $(\psi, \bar{\psi})$ the identity operator (1) and the energy density ϵ. Also the spin operator (σ) and the disorder field (μ) can be analysed by the counting method, but they are semi-local operators with respect to the asymptotic states. The magnetic perturbation breaks the Z_2 symmetry of the model and only scalar operators appear in this perturbation, namely the identity (1), the energy density (ϵ) and the spin operator (σ).

A general feature of the counting method is that the space of operators is determined by fermionic sum expressions. Such expressions also appear in the analysis of corner transfer matrices [7] and spinon conformal field theories [8]. It would be interesting to establish a more direct connection between these methods and the form factor approach. Finally note that the above example only constitutes a simple application of the counting method. It can be generalised to many other systems, including models with a massless spectrum and/or bounderies.

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